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FAST ALGEBRAIC METHODS FOR MULTIVARIABLE CONTROL SYSTEMS

Final Report

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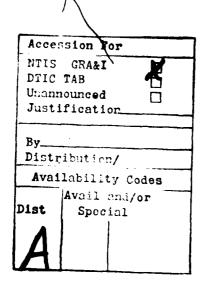
ABSTRACT (Continue on reverse aids if reservoiry and identify by block marker)

>> Several fast algebraic methods have been developed for the analysis and synthesis of multivariable control systems. Simple and quick methods based on the block-pulse functions and the matrix continued fractions have been derived for rapid model reduction, system identification, models conversion, rational matrix realization, and design of multivariable control systems. Also, computer-aided methods have been developed for determining the solvents, spectral factors of matrix polynomials and for evaluating the block partial fraction.

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expansions of rational matrices. Furthermore, a quick algorithm based on the matrix sign function has been developed for rapid solution of a matrix Riccati equation. The algorithm has successfully been applied to develop a fast suboptimal state-space self-tuner for linear stochastic multivariable systems.





Summary of Research Results

Several fast algebraic methods have been developed for the analysis and synthesis of multivariable control systems. Simple and quick methods based on the block-pulse functions and the matrix continued fractions have been derived for rapid model reduction, system identification, models conversion, rational matrix realization, and design of multivariable control systems. Also, computer-aided methods have been developed for determining the solvents, spectral factors of matrix polynomials and for evaluating the block partial fraction expansions of rational matrices. Furthermore, a quick algorithm based on the matrix sign function has been developed for rapid solution of a matrix Riccati equation. The algorithm has successfully been applied to develop a fast suboptimal state-space self-tuner for linear stochastic multivariable systems.

Main results of this research are described as follows:

1. Shieh, L.S., H. Wang and R.E. Yates, "Discrete-Continuous Model Conversion," <u>Applied Mathematical Modeling</u>, IPC Science and Technology Press, Vol. 4, pp. 449-455, December 1980.

<u>Abstract</u>

This paper presents methods for model conversions of continuous-time state-space equations and discrete-time state-space equations. An improved geometric-series method is presented for converting continuous-time models to equivalent discrete-time models. Also, a direct truncation method, a matrix continued fraction method and a geometric-series method are presented for converting discrete models to equivalent continuous models. As a result, many well-developed theorems and methods in

either continuous or discrete domains can be effectively applied to a suitable model in either domain.

2. Shieh, L.S., Y.J. Wei and R.E. Yates, "A Modified Direct-Decoupling Method for Multivariable Control System Designs," IEEE Trans. on Industrial Electronics and Control Instrumentation, Vol. IECI-28, No. 1, pp. 1-9, February 1981.

Abstract

A design method, which decouples an interactive system by using a compensator obtained from the plant inverse matrix, which is often called the direct-decoupling method, is modified in this paper. The modified direct-decoupling method uses the adjoint matrix instead of the inverse of the plant matrix to construct the compensator. The method uses a frequency-domain model-reduction method to simplify the degree of the given plant transfer function matrix and the obtained compensator. For an open-loop stable multivariable system, the proposed method gives a simple, practical, and realizable controller without using an unstable pole-zero cancellation approach.

3. Shieh, L.S. and N. Chahin, "A Computer-Aided Method for the Factorization of Matrix Polynomials," <u>International Journal of Systems Science</u>, Vol. 12, No. 3, pp. 305-323, March 1981.

Abstract

An extended multidimensional Newton-Raphson method is proposed for the factorization of matrix polynomials. A root-locus approach and a matrix continued fraction approach are presented to make initial guesses for rapid convergence of the Newton-Raphson method. The computer-aided method can be used to determine the spectral factors of a matrix polynomial for the

analysis and synthesis of kinematic and dynamic systems, and to obtain the spectral factorization of a matrix polynomial for optimal control and filtering problems. The same approach can be applied to determine the nth root of a real or complex matrix.

4. Tsay, Y.T. and L.S. Shieh, "A State-Space Approach for Self-Tuning Feedback Control with Pole Assignment," Proceedings of the IEE (England) Part D. Control Theory and Applications, Vol. 128, No. 3, pp. 93-101, May 1981.

<u>Abstract</u>

The paper presents a state-space approach for dealing with self-tuning control problems. A joint algorithm for system-parameter identification and system-state estimation is derived. With the joint algorithm, modern control techniques can be easily applied to systems having stochastic disturbances. The state-space version of the self-tuning control with pole assignment is investigated. Based on the state-space approach, the proposed self-tuning control algorithm can be applied to time-invariant linear systems including the unstable and nonminimum-phase systems.

5. Shieh, L.S., S. Yeh and H.Y. Zhang, "Realizations of Matrix Transfer Functions Using RC Ladders and Summers," <u>Proceedings of the IEE (England) Part G. Electronic Circuits and Systems</u>, Vol. 128, No. 3, pp. 101-110, June 1981.

Abstract

The design of a multivariable (multi-input/multi-output) control system or a multiport network often results in a multivariable compensator or a multiport filter which is described by a transfer-function matrix or a matrix transfer function. A

state-space approach is presented to synthesize the matrix transfer function, using RC ladders and summers but not using integrators and multiwinding transformers. The Cauer I type RC ladder network is utilized to construct accessible states at which the feedback and feedforward gains can be practically implemented. A new similarity transformation matrix is derived to transform a state-space equation in a block companion form to a state-space equation in a block tridiagonal form.

6. Shieh, L.S., S. Yeh and R.E. Yates, "Realizations of Matrix Transfer Functions Via a State-Space Approach," <u>IEEE Trans. on Circuits and Systems</u>, Vol. CAS-28, No. 9, pp. 934-939, September 1981.

Abstract

A state-space technique is presented for minimal realization of an impedance matrix or a voltage transfer function matrix without using integrators or the combination of integrators and multiwinding transformers. The Cauer II type passive RC ladder multiport and state-space concepts are employed for the realization. A new similarity transformation matrix is derived to transform a state-space equation in a RC-type Cauer second form into a state-space equation in a block companion form.

7. Shieh, L.S., Y.F. Chang and R.E. Yates, "A Dominant Data Matching Method for Digital Control Systems Modeling and Design," IEEE Trans. Industrial Electronics and Control Instrumentation, Vol. IEC1-28, No. 4, pp. 390-396, November.

Abstract

A dominant-data matching method is presented for obtaining a reduced-order pulse-transfer function from either a high-order continuous-data transfer function or a high-order pulse-transfer

function, and for identifying the pulse-transfer function of a system from available experimental time and frequency response data. The method may also be applied to the digital control systems design problem with various sampling periods.

8. Shieh, L.S. and Y.T. Tsay, "Transformations of Solvents and Spectral Factors of Matrix Polynomials, and their Applications," International Journal of Control, Vol. 34, No. 4, pp. 813-823, October 1981.

Abstract

The relationships between solvents and spectral factors of a high-degree matrix polynomial are explored. Various new transformations are developed to convert right (left) solvents into spectral factors and vice versa. The transformation of a right (left) solvent to a left (right) solvent is also established. The newly established algorithms are then applied to determine the spectral factorization of a matrix polynomial for optimal control problems. The developed algebraic theory enhances the capability of the analysis and synthesis of a system described by a high-degree matrix differential equation.

9. Shieh, L.S., Y.T. Tsay and N.P. Coleman, "Algorithms for Solvents and Spectral Factors of Matrix Polynomials," <u>International Journal of Systems Science</u>, Vol. 12, No. 11, pp. 1303-1316, November 1981.

Abstract

A generalized Newton method, based on the contracted gradient of a matrix polynomial, is derived for solving the right (left) solvents and spectral factors of matrix polynomials. Two methods of selecting initial estimates for rapid convergence of

the newly developed numerical method are proposed. Also, new algorithms for solving complete sets of the right (left) solvents and spectral factors without directly using the eigenvalues of matrix polynomials are derived. The proposed computer-aided method can be used to determine the spectral factorization of a matrix polynomial for optimal control, filtering and estimation problems.

10. Zhang, H.Y., L.S. Shieh and R.E. Yates, "Identification and Model Reduction of Multivariable Continuous Systems Via a Block-Pulse Functions Scheme," <u>Applied Mathematical Modelling</u>, IPC Science and Technology Press, Vol. 6, No. 5, pp. 369-373, October 1982.

Abstract

This article deals with the identification and model reduction of multi-input and multi-output (MIMO) continuous systems described by a high-degree vector differential equation or a matrix transfer function, from the discretized input and output data. The approach is based on using the block-pulse functions scheme and the least squares method. A two-shaft gas turbine model is used as an illustration.

11. Tsay, Y.T., L.S. Shieh, R.E. Yates and S. Barnett, "Block Partial Fraction Expansion of a Rational Matrix," <u>Linear and Multilinear Algebra</u>, Vol. 11, No. 3, pp. 225-241, March 1982.

Abstract

The main contribution is a block partial fraction expansion of a rational matrix $F(\lambda)$, $F(\lambda) = N_r(\lambda)D_r^{-1}(\lambda) = D_1^{-1}(\lambda)N_1(\lambda)$ and $N_r(\lambda)$, $D_r(\lambda)$, $D_1(\lambda)$ and $N_1(\lambda)$ are polynomial matrices. A new algorithm is derived to construct a transformation matrix that transforms a right (left) solvent to the corresponding left

(right) solvent of a matrix polynomial. Also, the algorithm can be used to construct a set of right (left) fundamental matrix polynomials and the inversion of a block Vandermonde matrix. This leads to a new technique to perform the block partial fraction expansion of a class of rational matrices.

 Shieh, L.S. and Y.T. Tsay, "Transformations of a Class of Multivariable Control Systems to Block Companion Forms," <u>IEEE</u> <u>Trans. Automatic Control</u>, Vol. AC-27, pp. 199-203, February 1982.

Abstract

Similarity block transformations are developed to transform a class of linear time-invariant MIMO state equations in arbitrary coordinates into block companion forms so that the classical lines of thought for SISO systems can be extended to MIMO systems.

13. Shieh, L.S. and Y.T. Tsay, "Block Modal Matrices and their Applications to Multivariable Control Systems," <u>Proceedings of the IEE (England) Part D. Control Theory and Applications</u>, Vol. 129, No. 2, pp. 41-48, March 1982.

<u>Abstract</u>

A left (right) block modal matrix is constructed to decompose a class of linear time-invariant MIMO state equations in arbitrary co-ordinates into a block diagonal canonical form Contains which left (right) solvents of a characteristic polynomial matrix. The applications of block modal matrices to block partial fraction expansions, analytical time-response solutions, and model reductions of high-degree matrix fraction descriptions and high-order state equations is examined. Also, the relationship between a left , gh. solvent of a matrix polynimial and the

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corresponding right (left) solvent is explored. The established block canonical forms in the time domain, and developed algebraic theories, provide additional mathematical tools for the analysis and synthesis of a class of MIMO control systems and matrix functions.

14. Tsay, Y.T. and L.S. Shieh, "Some Applications of Rational Matrices to Problems in Systems Theory," <u>International Journal of Systems Science</u>, Vol. 13, No. 12, pp. 1319-1337, December 1982.

<u>Abstract</u>

This paper deals with the residue theorems of rational λ -matrices having complex variables (λ) and the associated rational matrix functions with square matrix variables. First, some fundamental properties of λ -matrices and the associated matrix polynomials with square matrix variables are defined by employing the Cauchy's integral theorem. Then, a new matrix residue theorem is derived via the generalized Cauchy's integral theorem. Finally, the block partial fraction expansion and spectral factorization of rational λ -matrices are developed via the newly extended matrix residue theorem.

15. Tsay, Y.T. and L.S. Shieh, "Irreducible Divisors of Lambda-Matrices and their Applications to Multivariable Control Systems," International Journal of Control, Vol. 37, No. 1, pp. 17-36, January 1983.

Abstract

A new similarity block transformation is derived to transform state equations of multi-input-multi-output (MIMO) systems in general coordinates to those of cascade or parallel block canonical forms so that irreducible divisors of the

characteristic λ -matrix can be determined. Using the obtained complete set of irreducible divisors of the λ -matrix, an extended block partial fraction expansion of the matrix fraction description of a MIMO system is developed. Applications of the developed results to the model reduction and the suboptimal block controller design for high order systems are illustrated.

 Tsay, Y.T. and L.S. Shieh, "Block Decompositions and Block Modal Controls of Multivariable Control Systems," <u>Automatica</u>, Vol. 19, No. 1, pp. 29-40, January 1983.

<u>Abstract</u>

This paper concerns the properties and applications of two basic block canonical forms; the block companion form and the block diagonal form. A new scheme for determining the solvents of a λ -matrix and the block poles of rational λ -matrix from a class of state equations is presented. Also, a method for performing the block partial-fraction expansion of a matrix-fraction description from the block diagonalized state equation is derived. Furthermore, a new block modal controller via a cascade block companion form and block-poles assignment concept is developed for state-feedback controls of MIMO systems.

17. Shieh, L.S., Y.T. Tsay and R.E. Yates, "Some Properties of Matrix Sign Functions Derived from Continued Fractions," <u>Proceedings of the IEE (England) Part D. Control Theory and Applications</u>, Vol. 130, No. 3, pp. 111-118, May 1983.

Abstract

This paper proposes an alternate representation of a matrix sign function based on an irrational function described by a continued fraction. The properties of the continued fraction and

the truncated continued fractions are investigated. Also, new algorithms for computing the matrix sign function are developed. The matrix sign function is then extended to a generalized matrix sign function for directly solving discrete-time system problems.

18. Shieh, L.S., Y.T. Tsay and R.E. Yates, "State-Feedback Decomposition of Multivariable Systems Via Block-pole Placement," IEEE Trans. Automatic Control, Vol. AC-28, No. 8, pp., Aug. 1983.

Abstract

Some recently developed algebraic theorems of λ -matrices and applications to matrix fraction descriptions of a class of multivariable systems are introduced. Also, a new block-pole placement is developed for the state-feedback block decomposition of multivariable systems.

19. Shieh, L.S., C.T. Wang, and Y.T. Tsay, "Fast Suboptimal State-Space Self-tuner for Linear Stochastic Multivariable Systems," Proceedings of the IEE (England) Part D. Control Theory and Applications, Vol. 130, No. 4, pp., July 1983.

Abstract

A fast state-space self-tuner is developed for suboptimal control of linear stochastic multivariable systems. The suboptimal self-tuner is determined by utilizing both the standard recursive-extended-least-squares parameter estimation algorithm and the recently developed matrix sign algorithm, which gives a fast solution of the steady-state discrete Riccati equation. The developed suboptimal state-space self-tuner can be applied to a class of stable/unstable and minimum/non-minimum phase linear stochastic multivariable systems, in which the pair (A,C) is block observable and the pair (A,B) is stabilizable. Also, the pair

 $(A, \hat{\mathbb{Q}})$ with $\hat{\mathbb{Q}}\hat{\mathbb{Q}}^T = \mathbb{Q}$ is detectable where A,B and C are system, input and output matrices, respectively, and Q is a weighting matrix in a quadratic performance index.

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